

2022 MANITOBA MATHEMATICAL COMPETITION

(for students in grade 12)

Tuesday February 23, 2022, 9 – 11 AM

Sponsors:



UNIVERSITY
OF MANITOBA



- Mr.
 Mrs.
 Ms.

↑ Student's Name (print). Underline the Surname ↑

↑ Student's Signature ↑

↑ Student's School ↑

↑ Student's Home Street Address (or P. O. Box #) ↑

↑ City/Town

Postal Code ↑

Instructions for participants: Before the contest begins, complete the above information. Put no personal identifying information on any other pages. You should have received 12 pages in total, including this page, all showing the same paper number.

Answer each question on the page where it appears. If you run out of space below a question work may be continued on the back of the same page; if that is insufficient you may further continue work on blank pages 23 and 24.

No for-credit work should appear on the back of this cover page (it may be used for scrap).

Don't refer in any solution to work done on other questions; they are marked independently.

No aids are permitted—no straight edges, compasses or other mechanical drawing devices, electronics (cell phones, electronic watches, translators, tablets, calculators etc.).

This space may be used for scratch work. Do not continue solutions on this page—no credit will be given for work appearing here.

Question 1

- (a) Find all numbers x , if any, having the property that x percent of x is equal to x .
- (b) In a class of 11 students every mark (always an integer) from (some integer) n to $n + 10$ inclusive was attained by at least one student on the final exam. The total number of marks obtained in the class was 858. What was the highest mark in the class?

This space may be used to continue your solution for Question 1; it may be further continued on page 23 or 24—see instructions on page 23.

Question 2

- (a) Given that the positive quantities I, E, R, r and n are related by the formula

$$I = \frac{E}{R + \frac{r}{n}},$$

express n in terms of I, E, R and r .

- (b) Find all points, if any, where the curves $xy + 2x + 3y + 6 = 0$ and $x - y = 0$ intersect.

This space may be used to continue your solution for Question 2; it may be further continued on page 23 or 24—see instructions on page 23.

Question 3

(a) A small high school has exactly 100 students and offers second language courses in French, German and Spanish. Further,

- 30 students are enrolled in Spanish.
- 40 students are enrolled in German.
- 50 students are enrolled in French.
- 30 students are enrolled in both French and German.
- No students are enrolled in both French and Spanish.
- No students are enrolled in both German and Spanish.

How many of the 100 students are enrolled in none of the three language courses?

(b) A line through the origin $(0,0)$ in the xy -plane bisects the quadrilateral with vertices at $(1,1), (5,1), (1,2), (5,2)$ into two parts of equal area. Give an equation for that line.

This space may be used to continue your solution for Question 3; it may be further continued on page 23 or 24—see instructions on page 23.

Question 4

- (a) Find a pair of numbers x, y whose difference, sum, and product are in the ratio $2 : 7 : 45$.
- (b) A snowplow travels at 30 km/hr clearing the highway from Winnipeg to Brandon, then drives back to Winnipeg at 60 km/hr. What is the average driving speed of the vehicle over the complete round trip?

This space may be used to continue your solution for Question 4; it may be further continued on page 23 or 24—see instructions on page 23.

Question 5

- (a) A piece of string is randomly cut into two pieces. What is the probability that the longer piece is at least five times as long as the shorter piece?
- (b) Three children each choose a number (an integer) between 1 and 5 inclusive. What is the probability that exactly two children choose the same number?

This space may be used to continue your solution for Question 5; it may be further continued on page 23 or 24—see instructions on page 23.

Question 6

A roll of tape consists of a plastic spool on which is rolled a strip of tape that is 200 cm long, 1.5 cm wide, and 0.1 cm thick. The plastic spool has a radius of 1 cm. What is the radius of the entire roll, consisting of both spool and tape? (Leave your answer in terms of π .)

This space may be used to continue your solution for Question 6; it may be further continued on page 23 or 24—see instructions on page 23.

Question 7

Three vertices chosen from a regular n sided polygon form a triangle with angles of 96° , 54° , and 30° . Find, with proof, the minimum possible value for n .

(A *regular* n -sided polygon has n equal sides and n equal angles.)

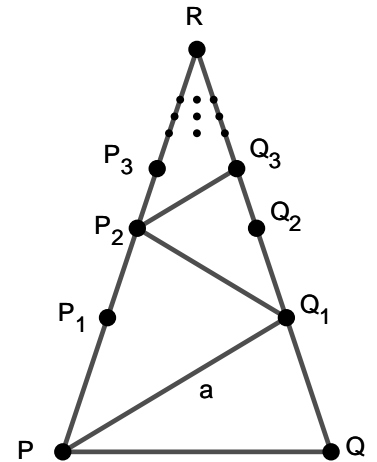
This space may be used to continue your solution for Question 7; it may be further continued on page 23 or 24—see instructions on page 23.

Question 8

PQ is the base of an isosceles triangle $\triangle PQR$. Define the infinite sequences of points $P_0 = P, P_1, P_2, \dots$ on the segment PR and Q_0, Q_1, Q_2, \dots on the segment QR so that for each $n \geq 0$, $\triangle P_n Q_n R$ is isosceles with base $P_n Q_n$, and such that for each $n \geq 1$, $P_n R : P_{n+1} R = r$, where $r > 1$ is fixed. Suppose $P_0 Q_1$ has length a . Show that the length of the zigzag path (approaching R via an infinite number of segments)

$$P_0 \rightarrow Q_1 \rightarrow P_2 \rightarrow Q_3 \rightarrow \dots$$

depends only on the values of a and r . Provide a formula for this length in terms of a and r .



This space may be used to continue your solution for Question 8; it may be further continued on page 23 or 24—see instructions on page 23.

Question 9

Find all prime numbers of the form $\frac{n(n+1)(n+2)}{6} + 1$ where n is a positive integer.

This space may be used to continue your solution for Question 9; it may be further continued on page 23 or 24—see instructions on page 23.

Question 10

A, B, C, D are distinct points on a circle of radius 5 with center O such that chords AB, BC and CD all have the same length, 8.

K is the point of intersection of lines AB and CD . What is the distance from O to K ?

(Your final solution may be in any form though the most simplified answers will receive the most credit. Partial credit will be awarded to well-organized and clear attempts making significant progress toward the answer.)

This space may be used to continue your solution for Question 10; it may be further continued on page 23 or 24—see instructions on page 23.

Both sides of this sheet may be used for continuation of solutions or for scratch work.

To receive credit for work continued here:

1. Clearly indicate in your solution that it is continued here.
2. Clearly indicate here which question is being continued (e.g., "Q7 (cont.)").
3. Clearly separate continued work from different questions and from scratch calculations.

This space may be used for scratch work, or to continue solutions—see instructions on page 23